Dear Kathy,

I hope the following is correct, and also that is has something to do with what you asked.

I don't think that in general the result the two convolutions is the same as a single convolution with a function of the same form. However, I think you can see what you do get.

I'll use the following notation. Let $\chi(t)$ be the original square wave. Thus

$$\chi(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t \geq 0. \end{cases}$$

Also, for any parameter $a > 0$ we will let $E_a(t) = ae^{-at}$ for $t > 0$.[1] Then if I understand correctly, you want to first convolve χ with E_a, and then convolve the result with E_b. (In your example, $a = \frac{1}{100}$ and $b = \frac{1}{300}$). If f and g are two functions, I'll write the convolution as

$$f * g(x) = \int_0^\infty f(t)\, g(x-t)\, dt.$$

Now

$$E_a * \chi(x) = \int_0^\infty E_a(t)\, \chi(x-t)\, dt = a \int_0^x e^{-at}\, dt = 1 - e^{-ax}.$$

This looks like your blue curve, I hope! Also, this tells you what the result of convolving the square wave once with any function E_a **must** look like.

Next we convolve this result with E_b. We get

$$\begin{aligned}
E_b * (E_a * \chi)(x) &= \int_0^x E_b(t)(E_a * \chi)(x-t)\, dt \\
&= \int_0^x be^{-bt}\left(1 - e^{-a(x-t)}\right) dt && \text{(since we need to keep } x - t \geq 0\text{)} \\
&= b \int_0^x e^{-bt}(1 - e^{-ax}e^{at})\, dt \\
&= b \int_0^x e^{-bt}\, dt - be^{-ax} \int_0^x e^{-(b-a)t}\, dt \\
&= 1 - e^{-bx} - be^{-ax}\left[\frac{1 - e^{-(b-a)x}}{b-a}\right] \\
&= 1 - \frac{be^{-ax} - ae^{-bx}}{b-a} && \text{after some algebra.}
\end{aligned}$$

This does not have the form $1 - e^{-cx}$ for any value of c, and so it seems to me that the red curve is not obtained from the square wave with a single convolution.

[1] I think you need to multiply e^{-at} by a, for otherwise you will not be convolving with a function with total integral equal to 1.

However, we can see what we are convolving with. We only need to find out what $E_b * E_a$ is. But

$$\begin{aligned}
E_b * E_a(x) &= \int_0^x E_b(t)E_a(x-t)\,dt \\
&= ab\int_0^x e^{-bt}e^{-a(x-t)}\,dt \\
&= abe^{-ax}\int_0^x e^{-(b-a)t}\,dt \\
&= \frac{ab}{b-a}e^{-ax}\left[1-e^{-(b-a)x}\right] \\
&= \frac{ab}{b-a}\left[e^{-ax}-e^{-bx}\right] \\
&= \frac{b}{b-a}E_a(x) \;-\; \frac{a}{b-a}E_b(x).
\end{aligned}$$

Thus the effect of convolving twice is the same as convolving with a linear combination of the two terms.

Also, if we take $b = a$, we get $E_a * E_a(x) = a^2xe^{-ax}$.